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What the Infrared Behaviour of QCD Vertex Functions in Landau gauge can tell us about Confinement

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The infrared behaviour of Landau gauge QCD vertex functions is investigated employing a skeleton expansion of the Dyson-Schwinger and Renormalization Group equations. Results for the ghost-gluon, three-gluon, four-gluon and quark-gluon vertex functions are presented. Positivity violation of the gluon propagator, and thus gluon confinement, is demonstrated. Results of the Dyson-Schwinger equations for a finite volume are compared to corresponding lattice data. It is analytically demonstrated that a linear rising potential between heavy quarks can be generated by infrared singularities in the dressed quark-gluon vertex. The selfconsistent mechanism that generates these singularities necessarily entails the scalar Dirac amplitudes of the full vertex and the quark propagator. These can only be present when chiral symmetry is broken, either explicitly or dynamically.

1. On theories of Quark Confinement

Quark confinement is definitely the hardest problem in hadron physics. Over the last three decades many theories have been suggested to elucidate this phenomenon. It has turned out that the main challenge for such theories is posed by the properties of the linearly rising static quark-antiquark potential as uncovered by many Monte-Carlo lattice calculations, see *e.g.* ref. ¹ and references therein for a corresponding discussion.

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Theories of confinement which are currently debated include ones based on

- (i) the condensation of chromomagnetic monopoles ^{2,3},
- (ii) the percolation of center vortices ⁴,
- (iii) the Gribov-Zwanziger scenario in Coulomb gauge ^{5,6}
- (iv) the infrared behaviour of Landau gauge Greens functions ^{7,8,9}, and
- (v) the AdS₅ / QCD correspondence ¹⁰.

Although at first sight these explanations for confinement are seemingly different there are surprising relations between them which are not yet understood. With the present level of understanding one has to note that these theories are definitely not mutually exclusive but simply reveal only different aspects of the confinement phenomenon.

In this talk I will focus on what one can learn from the infrared behaviour of Landau gauge Greens functions about confinement.^a

2. Infrared Structure of Landau gauge Yang-Mills theory

The starting point for our considerations is the Dyson-Schwinger equation for the ghost-gluon vertex as depicted in fig. 1. In the Landau gauge the gluon propagator is transverse, and thus one has the relation

$$l_\mu D_{\mu\nu}(l - q) = q_\mu D_{\mu\nu}(l - q), \quad (1)$$

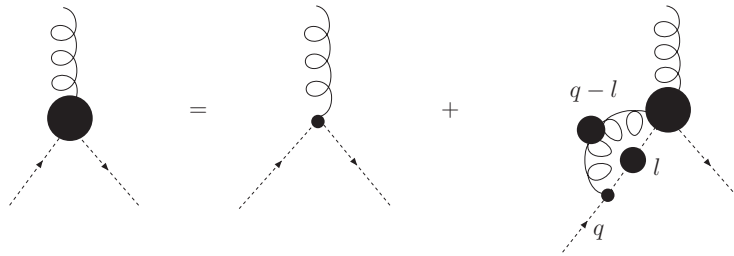


Fig. 1. The Dyson-Schwinger equation for the ghost-gluon vertex.

which immediately allows one to conclude that the ghost-gluon vertex stays finite when the outgoing ghost momentum vanishes, *i.e.* when $q_\mu \rightarrow 0$ ¹². This argument is valid to all orders in perturbation theory, a truly non-perturbative justification of the related infrared finiteness has been given in refs. ^{13,14,15}.

This property of the ghost-gluon vertex makes the Dyson-Schwinger equation for the ghost propagator, see fig. 2, tractable. The only unknowns in the deep

^aSee also the recent review on this and related issues given in ref. ¹¹.

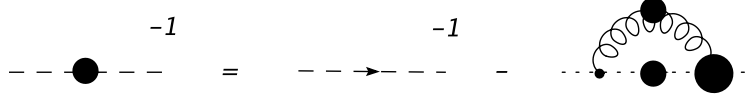


Fig. 2. The Dyson-Schwinger equation for the ghost propagator.

infrared are the gluon and the ghost propagators. In Landau gauge these propagators are parametrized by two invariant functions, denoted here $Z(k^2)$ and $G(k^2)$, respectively. In Euclidean momentum space one has

$$D_{\mu\nu}(k) = \frac{Z(k^2)}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right), \quad D_G(k) = -\frac{G(k^2)}{k^2}. \quad (2)$$

After renormalization the functions $Z(k^2)$ and $G(k^2)$ depend also on the renormalization scale μ . Furthermore, assuming that the QCD Green functions can be expanded in asymptotic series, the integral in the ghost Dyson-Schwinger equation can be split up in three pieces, an infrared integral, an ultraviolet integral, and an expression for the ghost wave function renormalization. As a matter of fact, it is the resulting equation for the latter quantity which allows one to extract definite information¹⁶ without using any truncation or ansatz.

It turns out that the infrared behaviour of the gluon and ghost propagators is given by power laws, and that the exponents are uniquely related such that the gluon exponent is -2 times the ghost exponent¹⁷. As we will see later on this implies an infrared fixed point for the corresponding running coupling. The signs of the exponents are such that the gluon propagator is infrared suppressed as compared to the one for a free particle, the ghost propagator is infrared enhanced.

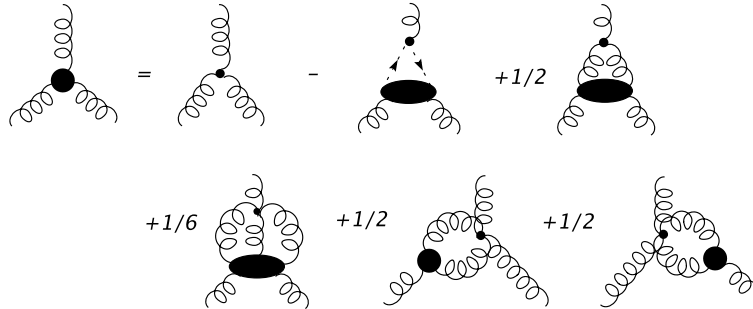


Fig. 3. The Dyson-Schwinger equation for the 3-gluon vertex.

The fact that the Yang-Mills propagators obey infrared power laws can be employed to study the infrared behaviour of higher n -point functions. To this end the

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corresponding n -point Dyson-Schwinger equations have been studied in a skeleton expansion, *i.e.* a loop expansion using dressed propagators and vertices. Furthermore, an asymptotic expansion has been applied to all primitively divergent Green functions¹⁸. As an example consider the Dyson-Schwinger equation for the 3-gluon vertex which is diagrammatically represented in fig. 3. Its skeleton expansion, see fig. 4, can be constructed via the insertions given in fig. 5. These insertions have

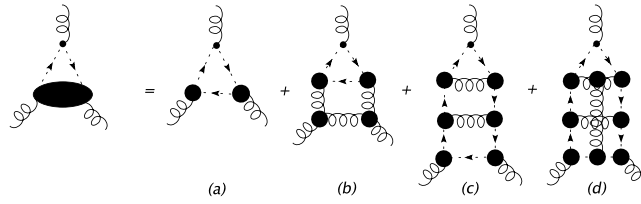


Fig. 4. An example for the skeleton expansion of the 3-gluon vertex.

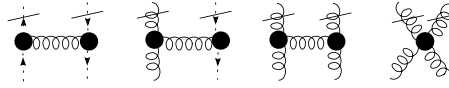


Fig. 5. Insertions to reconstruct higher orders in the skeleton expansion.

vanishing infrared anomalous dimensions which implies that the resulting higher order terms feature the same infrared scaling. Based on this the following general infrared behaviour for one-particle irreducible Green functions with $2n$ external ghost legs and m external gluon legs can be derived^{18,19}:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa+(1-n)(d/2-2)} \quad (3)$$

where κ is one yet undetermined parameter, and d is the space-time dimension.

Very recently it has been shown²⁰ by exploiting Dyson-Schwinger equations and Exact Renormalization Group Equations (*cf.* figs. 2 and 6 for the differences in these equations for the ghost propagator) that this infrared (IR) solution is unique. It especially includes that

- (i) the ghost propagator is IR divergent,
- (ii) the gluon propagator is IR suppressed,
- (iii) the ghost-gluon vertex is IR finite,
- (iv) the 3- and 4-gluon vertex are IR divergent,
- (v) the ghost sector dominates the IR, and
- (vi) every coupling from an Yang-Mills vertex possesses an IR fixed point,
i.e. Infrared Yang-Mills theory is conformal.

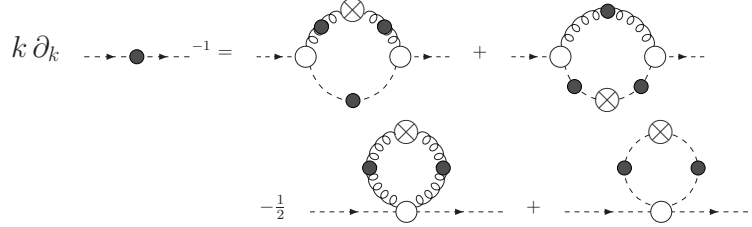


Fig. 6. The Exact Renormalization Group Equation for the ghost propagator.

3. Yang-Mills running coupling: Infrared fixed point

The infrared behaviour (3) especially includes

$$\begin{aligned} G(p^2) &\sim (p^2)^{-\kappa}, \quad Z(p^2) \sim (p^2)^{2\kappa} \\ \Gamma^{3g}(p^2) &\sim (p^2)^{-3\kappa}, \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa} \end{aligned} \quad (4)$$

which allows to conclude that the running couplings as inferred from these vertex functions possess an infrared fixed point:

$$\alpha^{gh-gl}(p^2) = \alpha_\mu G^2(p^2) Z(p^2) \sim \frac{\text{const}_{gh-gl}}{N_c}, \quad (5)$$

$$\alpha^{3g}(p^2) = \alpha_\mu [\Gamma^{3g}(p^2)]^2 Z^3(p^2) \sim \frac{\text{const}_{3g}}{N_c}, \quad (6)$$

$$\alpha^{4g}(p^2) = \alpha_\mu \Gamma^{4g}(p^2) Z^2(p^2) \sim \frac{\text{const}_{4g}}{N_c}. \quad (7)$$

In particular, the infrared value of the coupling related to the ghost-gluon vertex can be calculated^{13,21}:

$$\alpha^{gh-gl}(0) = \frac{4\pi}{6N_c} \frac{\Gamma(3-2\kappa)\Gamma(3+\kappa)\Gamma(1+\kappa)}{\Gamma^2(2-\kappa)\Gamma(2\kappa)} \quad (8)$$

which yields $\alpha^{gh-gl}(0) = 2.972$ for $N_c = 3$ and $\kappa = (93 - \sqrt{1201})/98 \simeq 0.595353$, which is the value obtained with a bare ghost-gluon vertex.

4. Positivity violation for the gluon propagator

Positivity violation of the (space-time) propagator of transverse gluons has been a long-standing conjecture for which there is now compelling evidence, see *e.g.* refs. ^{22,23} and references therein. The basic feature is the infrared suppression of transverse gluons caused by the infrared enhancement of ghost correlations. A simple argument given by Zwanziger makes this obvious: An infrared vanishing gluon propagator leads to a vanishing integral over the space-time gluon propagator,

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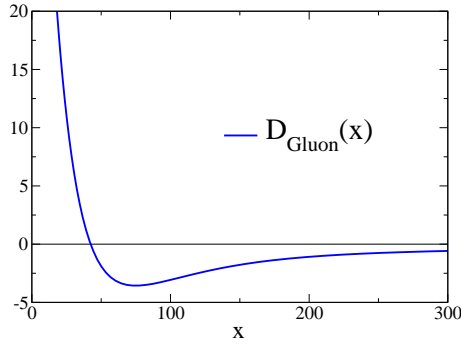


Fig. 7. The Fourier transform of the solution of the Dyson-Schwinger equation for the gluon propagator, *cf.* ref. 22.

the latter being the Fourier transform of the momentum space gluon propagator. Therefore one has

$$0 = D_{gluon}(k^2 = 0) = \int d^4x \quad D_{gluon}(x) \quad (9)$$

This implies that $D_{gluon}(x)$ has to be negative for some values of x . And, as a matter of fact this behaviour is seen from fig. 7 in which the Fourier transform of the result for the gluon propagator is displayed. As this behaviour clearly signals the confinement of transverse gluons²⁴ it is certainly worth to have a closer look at the underlying analytic structure of the gluon propagator.

To investigate the analytic structure of the gluon propagator we parameterize the running coupling such that the numerical results for Euclidean scales are quite accurately reproduced²⁵:

$$\alpha_{\text{fit}}(p^2) = \frac{\alpha_S(0)}{1 + p^2/\Lambda_{\text{qcd}}^2} + \frac{4\pi}{\beta_0} \frac{p^2}{\Lambda_{\text{qcd}}^2 + p^2} \left(\frac{1}{\ln(p^2/\Lambda_{\text{qcd}}^2)} - \frac{1}{p^2/\Lambda_{\text{qcd}}^2 - 1} \right) \quad (10)$$

with $\beta_0 = (11N_c - 2N_f)/3$. In this expression the Landau pole has been subtracted, it is analytic in the complex p^2 plane except the real timelike axis where the logarithm produces a cut for real $p^2 < 0$, and it obeys Cutkosky's rule.

The infrared exponent κ is an irrational number, and thus the gluon propagator possesses a cut on the negative real p^2 axis. It is possible to fit the solution for the gluon propagator quite accurately without introducing further singularities in the complex p^2 plane²²:

$$Z_{\text{fit}}(p^2) = w \left(\frac{p^2}{\Lambda_{\text{qcd}}^2 + p^2} \right)^{2\kappa} (\alpha_{\text{fit}}(p^2))^{-\gamma}. \quad (11)$$

Hereby w is a normalization parameter, and $\gamma = (-13N_c + 4N_f)/(22N_c - 4N_f)$ is the one-loop value for the anomalous dimension of the gluon propagator. The discontinuity of (11) along the cut vanishes for $p^2 \rightarrow 0^-$, diverges to $+\infty$ at $p^2 = -\Lambda_{\text{QCD}}^2$ and goes to zero for $p^2 \rightarrow \infty$.

The function (11) contains only four parameters: the overall magnitude which due to renormalization properties is arbitrary (it is determined via the choice of the renormalization scale), the scale Λ_{QCD} , the infrared exponent κ and the anomalous dimension of the gluon γ . The latter two are not free parameters: κ is determined from the infrared properties, and for γ the one-loop value is used. Thus one has found a parameterization of the gluon propagator which has effectively only one parameter, the scale Λ_{QCD} . It is important to note that the gluon propagator possesses a form such that *Wick rotation is possible!*

Note that positivity violation for gluons is also found at very high temperatures, even in the infinite temperature limit^{26,27,28}. For the gluons being transverse to the medium it applies in both phases. This does not come as a surprise: The infinite temperature limit corresponds to three-dimensional Yang-Mills theory plus an additional Higgs-type field inherited from the A_4 field. The latter decouples in the infrared, the three-dimensional Yang-Mills theory is as expected to be confining and thus the corresponding gluon modes are positivity violating. This especially entails a solution of the Linde problem²⁹, see *e.g.* ref.³⁰ and references therein. Based on this scheme it is safe to conclude that the static chromomagnetic sector is never deconfined, *cf.* ref.³¹.

5. Ghost and Glue in a box

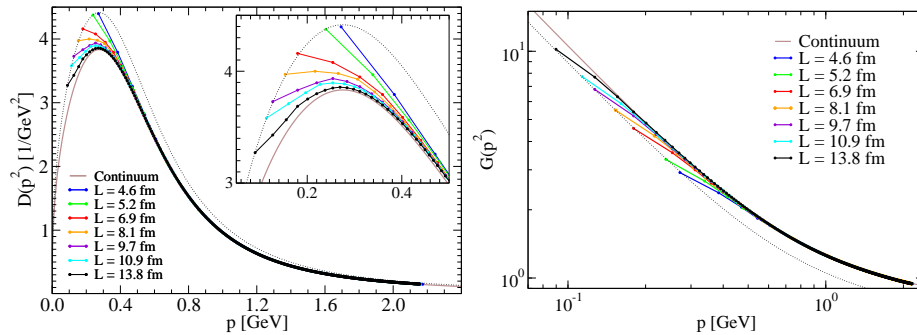


Fig. 8. The solution of the Dyson-Schwinger equation for the gluon and the ghost propagators at finite volumes (adapted from ref.³⁵).

As can be noted from refs.^{21,25} the numerical results for the ghost and gluon propagators compare very well to corresponding recent lattice data. However, the values of the infrared exponents extracted from lattice calculations do neither agree

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with the analytical obtained continuum results nor do they agree when compared against each other. A comparison to lattice calculations in three and two spacetime dimensions suggest that current lattice volumes are much too small for reliable extraction of infrared exponents^{32,33}.

At this point it is interesting to note that the Dyson-Schwinger equations can be solved on a compact manifold with finite volume^{34,35}. As can be seen from figs. 8 and 9 the approach to the continuum limit is quite slow.

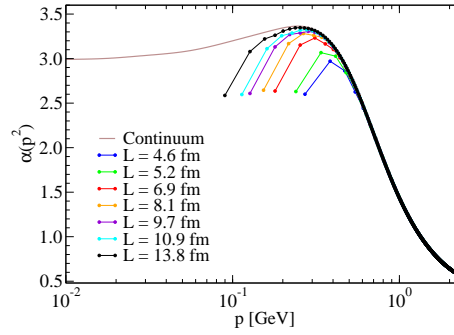


Fig. 9. The resulting running coupling at finite volumes (adapted from ref. ³⁵).

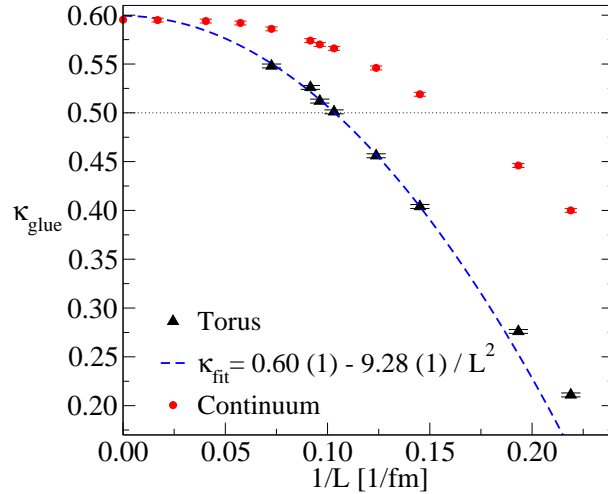


Fig. 10. The extracted value of the infrared exponent κ as function of the inverse of the torus length (adapted from ref. ³⁵).

This slow approach can be understood from fig. 10. The triangles represent the

torus results, the circles are the values of the infrared exponent κ as extracted from the continuum result when restricted to momenta $p \leq 2\pi/L$. This makes plain that for a precise extraction of the infrared exponent a large separation of scales is needed. This, and not the existence of a very small momentum scale inherent to the problem, is the reason why large volumes are needed for a precise determination of the infrared exponent.

In fig. 11 the results of the Dyson-Schwinger equations for the propagators are compared to recent lattice data³⁶ using very similar volumes. The agreement is, at least, satisfactory.

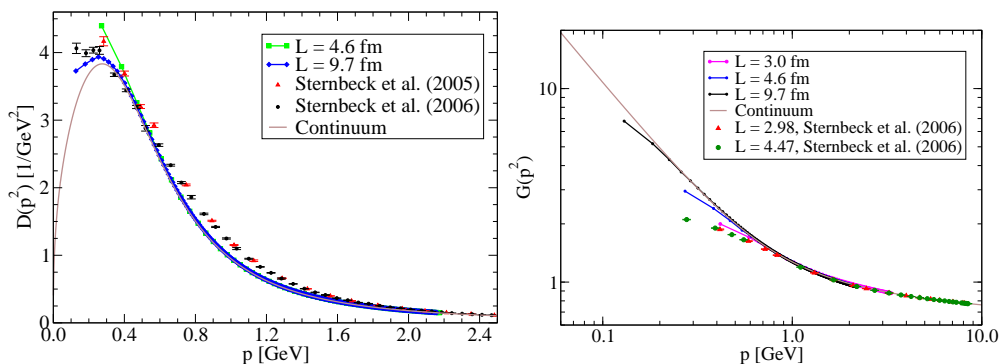


Fig. 11. The solution of the Dyson-Schwinger equations for the gluon and ghost propagators at finite volumes compared to lattice data (adapted from ref. ³⁵).

6. Dynamically induced scalar quark confinement

From the presentation above it is quite obvious how gluon confinement works in a covariant gauge. However, given the infrared suppression of the gluon propagator quark confinement is even seemingly more mysterious than ever. To proceed in the same spirit as above one studies the Dyson-Schwinger equation for the quark propagator. It turns out that the precise structure of the quark propagator depends crucially on the quark-gluon vertex^{37,7,8,25}. Therefore a detailed study of this three-point function, and especially its infrared behaviour, is mandatory. Its Dyson-Schwinger equation is diagrammatically depicted in fig. 12, its skeleton expansion in fig. 13.

At this point one has to notice a drastic difference of the quarks as compared to Yang-Mills fields: They possess a current, *i.e.* a tree-level, mass.^b To extend the infrared analysis of Yang-Mills theory described above to full QCD³⁸ we thus want

^bEven if this were not the case one expects dynamical chiral symmetry breaking and thus dynamical mass generation to occur.

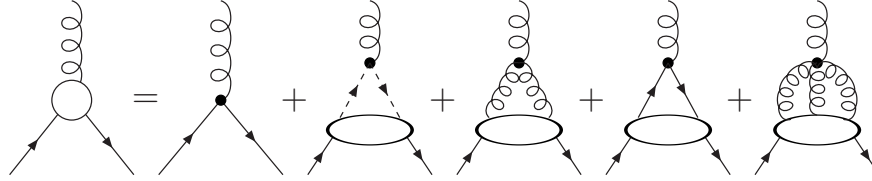
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Fig. 12. The Dyson-Schwinger equation for the quark-gluon vertex.

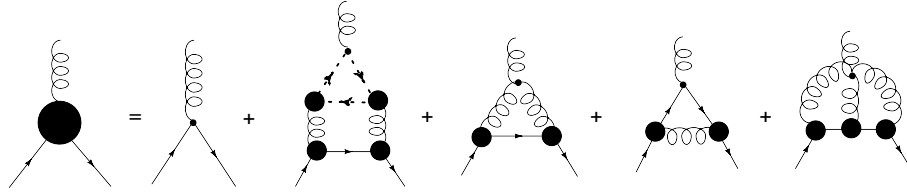


Fig. 13. The skeleton expansion for the quark-gluon vertex.

to concentrate first on the quark sector of quenched QCD and choose the masses of the valence quarks to be large, *i.e.* $m > \Lambda_{\text{QCD}}$. The remaining scales below Λ_{QCD} are those of the external momenta of the propagators and vertex functions. The relevant infrared limit is the one where all these external momenta approach zero. Then the Dyson-Schwinger equations can be used to determine the selfconsistent solutions in terms of powers of the small external momentum scale $p^2 \ll \Lambda_{\text{QCD}}$. The equations which have to be considered in addition to the ones of Yang-Mills theory are the one for the quark propagator and the quark-gluon vertex.

The dressed quark-gluon vertex Γ_μ consists in general of twelve linearly independent Dirac tensors. Some of those would be forbidden if chiral symmetry would be realized in the Wigner-Weyl mode. On the other hand, these tensor structures can be non-vanishing either if chiral symmetry is explicitly broken by current masses and/or chiral symmetry is realized in Nambu-Goldstone mode (*i.e.* spontaneously broken). From a solution of the Dyson-Schwinger equations we infer that these “Dirac-scalar” structures are, in the chiral limit, generated non-perturbatively together with the dynamical quark mass function in a self-consistent fashion: Dynamical chiral symmetry breaking reflects itself not only in the propagator but also in the quark-gluon vertex function.

An infrared analysis of the full set of Dyson-Schwinger equations reveals an infrared divergent solution for the quark-gluon vertex. Hereby, Dirac vector and “scalar” components of this vertex are infrared divergent with exponent $-\kappa - \frac{1}{2}$ ³⁸. A numerical solution of a truncated set of Dyson-Schwinger equations confirms this infrared behavior. Again, the diagrams containing ghost loops dominate. Thus all infrared effects from the Yang-Mills sector are generated by the infrared asymptotic

theory described above. More importantly, in the quark sector the driving pieces of this solution are the scalar Dirac amplitudes of the quark-gluon vertex and the scalar part of the quark propagator. Both pieces are only present when chiral symmetry is broken, either explicitly or dynamically.

For the coupling related to the quark-gluon vertex we obtain, using

$$\Gamma^{qg}(p^2) \sim (p^2)^{-1/2-\kappa}, \quad Z_f(p^2) \sim \text{const}, \quad Z(p^2) \sim (p^2)^{2\kappa}, \quad (12)$$

that

$$\alpha^{qg}(p^2) = \alpha_\mu [\Gamma^{qg}(p^2)]^2 [Z_f(p^2)]^2 Z(p^2) \sim \frac{\text{const}_{qg}}{N_c} \frac{1}{p^2}, \quad (13)$$

i.e. that it is singular in the infrared contrary to the couplings from the Yang-Mills vertices.

With similar methods one finds for the four-quark function an anomalous infrared exponent -2 . Note that the static quark potential can be obtained from this four-quark one-particle irreducible Greens function, which, including the canonical dimensions, behaves like $(p^2)^{-2}$ for $p^2 \rightarrow 0$. Therefore employing the well-known relation for a function $F \propto (p^2)^{-2}$ one obtains

$$V(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} F(p^0 = 0, \mathbf{p}) e^{i\mathbf{p}\mathbf{r}} \sim |\mathbf{r}| \quad (14)$$

for the static quark-antiquark potential $V(\mathbf{r})$. We conclude at this point that, given the infrared divergence of the quark-gluon vertex as found in the solution of the coupled system of Dyson-Schwinger equations, the vertex overcompensates the infrared suppression of the gluon propagator, and one therefore obtains a linear rising potential. In addition, this potential is dynamically induced and dominantly scalar.

However, there are two caveats. First, the uniqueness of this solution could not be shown. Second, because most of the terms in the skeleton expansion of the four-quark function are equally enhanced in the infrared the string tension could only be calculated by summing over an infinite number of diagrams. This property alleviates the usefulness of the approach but it had to be expected in the first place. Since already an effective, nonperturbative one-gluon exchange generates the confining potential one is confronted with the problem of unwanted van-der-Waals forces. To avoid such forces there has to occur a precise cancelation of them amongst the infinitely many terms contributing to the long-range part of the potential.

An interesting limit can, however, be studied. Suppose chiral symmetry is artificially kept in Wigner-Weyl mode, *i.e.* in the chiral limit we force the quark mass term as well as the “scalar” terms in the quark-gluon vertex to be zero. We then find $-\kappa$ as infrared exponent for the vertex, and the resulting running coupling from the quark-gluon vertex is no longer diverging but goes to a fixed point in the infrared similar to the couplings from the Yang-Mills vertices. Correspondingly, one obtains a constant for $\Gamma^{0,0,2}(p^2 = 0)$ and

$$V(\mathbf{r}) \sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2} e^{i\mathbf{p}\mathbf{r}} \sim \frac{1}{|\mathbf{r}|}.$$

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The “forced” restoration of chiral symmetry is therefore directly linked with the disappearance of quark confinement. The infrared properties of the quark-gluon vertex in the “unforced” solution thus constitute a novel mechanism that directly links chiral symmetry breaking with confinement.

7. Summary

To summarize the most important findings as inferred from the infrared analysis of all one-particle irreducible Green functions of Landau gauge QCD we note:

Gluons are confined by ghosts, and positivity of transverse gluons is violated.

The analytic structure of the resulting gluon propagator is such that Wick rotation is possible.

In the Yang-Mills sector the strong running coupling is infrared finite whereas the running coupling from the quark-gluon vertex is infrared divergent.

Chiral symmetry is dynamically broken, and this takes place in the quark propagator and the quark-gluon vertex.

We have provided evidence that static quark confinement in the Landau gauge is due to the infrared divergence of the quark-gluon vertex.

In the infrared this vertex is dominated by its scalar components thereby inducing a relation between confinement and broken chiral symmetry.

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